AD-A244 271

(2)

THE STROH FORMALISM FOR ANISOTROPIC ELASTICITY WITH APPLICATIONS TO COMPOSITE MATERIALS

FINAL REPORT

T. C. T. TING

OCTOBER 7, 1991

U. S. ARMY RESEARCH OFFICE

DAAL 03–88–K–0079

University of Illinois at Chicago
Department of Civil Engineering, Mechanics and Metallurgy
Box 4348, Chicago, IL 60680

MASTER COPY KEEP THIS COPY FOR REPRODUCTION PURPOSES

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
October 7, 1991

3. REPORT TYPE AND DATES COVERED
Final Report (6/15/1988 - 6/14/1991)

4. TITLE AND SUBTITLE
The Stroh formalism for anisotropic elasticity with applications to composite materials

5. FUNDING NUMBERS
DAAL03-88-K-0079

6. AUTHOR(S)
T. C. T. Ting

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Illinois at Chicago
CEMM Dept. (M/C246)
Box 4348, Chicago, IL 60680

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARO 25122.14-MA

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This is the final report for the research project entitled "The Stroh formalism for anisotropic elasticity with applications to composite materials". Thirteen papers have been published under this research project. Important findings of the results are outlined, and potential applications to composite materials are indicated.

14. SUBJECT TERMS

15. NUMBER OF PAGES
5

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

A. STATEMENT OF THE PROBLEM STUDIED

Anisotropic elasticity has been an active research topic since the need of high strength, light weight composites in aerospace industry became apparent. A composite material consists of two or more materials which are in general anisotropic. The oldest theory of two–dimensional anisotropic elasticity is due to Lekhnitskii. The Lekhnitskii theory is not only outdated, it is inefficient. A new theory, originally due to Stroh (1958, 1962) and further developed by others, is very powerful and elegant. We have extended the Stroh formalism by presenting new identities and sum rules. Using these identities and sum rules, some heretofore unsolved problems are solved and solutions which are available but are in a complex form are converted into a real form. With the solutions in a real form, many new physically interesting phenomena have been discovered. Some of these findings, such as the invariant properties of rotations about the x_3 axis, would be useful in design of composite materials.

B. SUMMARY OF THE MOST IMPORTANT RESULTS

Thirteen papers have been published under this research project. They are listed below under Section C.

The research findings can be divided into three categories:

(a) Re–interpretations of, and/or discovery of new phenomena from, existing solutions — [1, 5, 12].

(b) Extension of known solutions to include more general cases. — [2, 3, 7, 8, 10]

(c) Solutions to heretofore unsolved problems. — [1, 3, 4, 6 – 13].

Some of the papers cover more than one category.

Many anisotropic elasticity problems contain the three Barnett–Lothe (1973) tensors in their final solutions. These tensors are real and depend on material constants only. They can be expressed in an algebraic form in terms of complex eigenvalues and eigenvectors of the elasticity constants. They can also be expressed in an integral form directly in terms of the elasticity constants. The integral form provides a real expression. Explicit, closed form expressions of these tensors directly in terms of the elasticity constants were available only for isotopic materials and transversely isotropic materials. We obtained explicit expressions of the Barnett–Lothe tensors for orthotropic materials [4] and for monoclinic materials whose material symmetry plane is at $x_3 = 0$ [9].

The well–known Eshelby theorem says that, for an anisotropic elliptic inclusion in an infinite medium of different anisotropic material subject to a uniform loading at infinity, the stress inside the inclusion is uniform. However, the value of the uniform stress inside the inclusion has been determined only for special materials such as orthotropic materials. We obtained explicit expression of the stress inside the inclusion for general anisotropic elastic materials in the inclusion and the medium [3].

Applications of the Stroh formalism to composite materials are presented in [1. 2. 7. 12]. Interface cracks in composite materials are one of the actively studied problems in composites. We presented certain invariants relating to the re–orientation of the layers in the composite [7] and a new, concise solution which shows clearly that the surface tractions along the interface are polarized on an eigenplane while the crack opening displacements are polarized on a different eigenplane [12].

Interesting mathematical properties of certain quantities which appear in anisotropic elasticity as well as their physical significance are reported in [10, 13]. The classical paradox of Levy (1899) and Carothers (1912) on isotropic elastic wedge, which has been resolved by Dempsey (1981) and Ting (1984), has been extended for anisotropic elastic wedge (Ting, 1988) and the paradox for anisotropic elastic wedge is resolved in [6].

C. PUBLICATIONS UNDER THIS PROJECT

[1] T. C. T. Ting, "Recent advances in the theory of anisotropic elasticity with applications to composite materials," in Symp. on Recent Advances in Macro– and Micro–Mechanics of Composite Materials. David Hui and J. R. Vinson, eds. ASME AD–Vol. **13**, G00448, 205–212 (1988).

[2] Qianqian Li and T. C. T. Ting, "Line inclusions in anisotropic elastic solids," J. Appl. Mech. **56**, 556–563 (1989).

[3] Chyanbin Hwu and T. C. T. Ting, "Two–dimensional problems of the anisotropic elastic solid with an elliptic inclusion," Q. J. Mech. Appl. Math. **42**, 553–572 (1989).

[4] Changsong Dongye and T. C. T. Ting, "Explicit expressions of Barnett–Lothe tensors and their associated tensors for orthotropic materials," Q. Appl. Math. **47**, 723–734 (1989).

[5] T. C. T. Ting, "The eigenvectors of the **S** matrix and their relations with line dislocations and forces in anisotropic elastic solids," in Micromechanics and Inhomogeneity, The Toshio Mura Anniversary Volume. Springer–Verlag. N.Y. 449–467 (1990).

[6] Chyanbin Hwu and T. C. T. Ting, "Solutions for the anisotropic elastic wedges at critical wedge angles," J. Elasticity. **24**, 1–20 (1990).

[7] T. C. T. Ting, "Interface cracks in anisotropic bimaterials," J. Mech. Phys. Solids. **38**, 505–513 (1990).

[8] T. C. T. Ting and Gongpu Yan, "The anisotropic elastic solids with an elliptic hole or rigid inclusion," Int. J. Solids Structures. **27**, 1879–1894 (1991).

[9] T. C. T. Ting, "Barnett–Lothe tensors and their associated tensors for monoclinic materials with the symmetry plane at $x_3 = 0$," J. Elasticity. **27**, In press.

[10] T. C. T. Ting, "The Stroh formalism and certain invariances in two–dimensional anisotropic elasticity," in Modern Theory of Anisotropic Elasticity and Applications. J. J. Wu, T. C. T. Ting and D. M. Barnett, eds. SIAM Proceedings on Appl. Math. **57**. In press.

[11] T. C. T. Ting, "Image singularities of Green's functions for anisotropic elastic half–spaces and bimaterials," Q. J. Mech. Appl. Math. In press.

[12] T. C. T. Ting, "Interface cracks in anisotropic elastic bimaterials – a decomposition principle," The George Herrmann Symposium Volume, in Int. J. Solids Structures. In press.

[13] T. C. T. Ting, "On the orthogonal, Hermitian and positive definite properties of the matrices $i\mathbf{B}^{-1}\overline{\mathbf{B}}$ and $-i\mathbf{A}^{-1}\overline{\mathbf{A}}$ in anisotropic elasticity," J. Elasticity. In press.

D. PARTICIPATING SCIENTIFIC PERSONNEL ON THE PROJECT

Chyanbin Hwu, Ph. D.

Qianqian Li, Ph. D.

Changsong Dongye, Visiting Scholar, from Dalien Railway Institute, Dalien, China.

Gongpu Yan, M. S.

M. Z. Wang, Visiting Scholar, from Peking University, Beijing, China.

T. C. T. Ting, Principal Investigator.

BIBLIOGRAPHY

Barnett, D. M. and Lothe, J. (1973), "Synthesis of the sextic and the integral formalism for dislocations, Green's functions and surface waves in anisotropic elastic solids," Phys. Nor., **7**, 13–19.

Carothers, S. D. (1912), Proc. Roy. Soc. Edingburgh, **23**, 292.

Dempsey, J. D. (1981), "The wedge subjected to tractions: a paradox resolved," J. Elasticity, **11**, 1–10.

Lekhnitskii, S. G. (1981), Theory of Elasticity of an Anisotropic Body. MIR Pub., Moscow.

Levy, M. (1899), Comp. Rend., **126**, 1235.

Stroh, A. N. (1958), "Dislocations and cracks in anisotropic elasticity," Phil. Mag. **3**, 625–646.

Stroh, A. N. (1962), "Steady state problems in anisotropic elasticity," J. Math. Phys. **41**, 77–103.

Ting, T. C. T. (1984), "The wedge subject to tractions: A paradox re–examined," J. Elasticity, **14**, 235–247.

Ting, T. C. T. (1988), "The critical angle of the anisotropic elastic wedge subject to uniform tractions, J. Elasticity, **20**, 113–130.